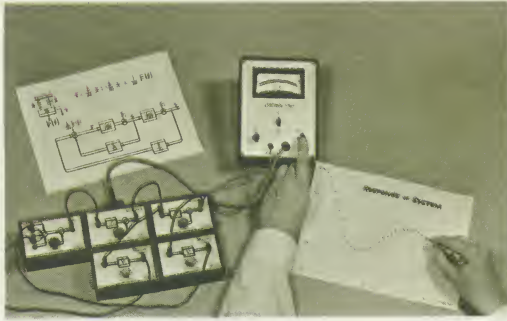




The Dynamic Analyzer.

A periodical devoted to applications of the Personal Analog Computer.

Published by Pastoriza Electronics, Inc. 285 Columbus Ave., Boston 16, Mass.

The Natural Manufacture of Sines and Cosines and Logarithms (with loops)

To the engineer exponential functions are an old and constantly recurring story. To the non scientist, the musician, the business man, the politician, they are a vague memory from grammar school days involving right triangle problems and multiplication of large numbers.

It is a remarkable thing, however, that all kinds of people encounter sines, cosines and logarithmic functions in their everyday life and scarcely recognize them, let alone appreciate why these functions occur. With the *Personal Analog Computer*, we can learn why.

Sines and Exponentials in Nature

Consider how often exponential functions occur in nature. We find that the growth of things, plants, organisms, populations is basically exponential. We find that things that wobble, musical instruments, the stock market, or poor automobile drivers approximate combinations of sine waves. Now, consider further the mathematical complexity of the exponential series:

$$e^{ct} = 1 + ct + \frac{c^2 t^2}{2!} + \frac{c^3 t^3}{3!} + \dots$$

One can hardly help wondering where it all ties in.

How can Nature do it?

With *Personal Analog Computer* units, we can generate time functions such as t , t^2 , and t^3 by successive integrations. Nature can do the same with her natural integrators. However, the exponential function seems to require an infinite number of integrations. How can nature do it without using an infinite number of integrators? How does nature's structure correspond to the structure of mathematics?

Most people are vague about the connection between mathematics and nature, because they either specialize in mathematics, and are aloof from nature, or they are practical fellows, who use mathematics only as one of many tools, usually as a more elegant articulation of what is already known. The analog computer straddles the two worlds and provides an insight into both.

Let us study a simple causal loop with the *Personal Analog Computer*.

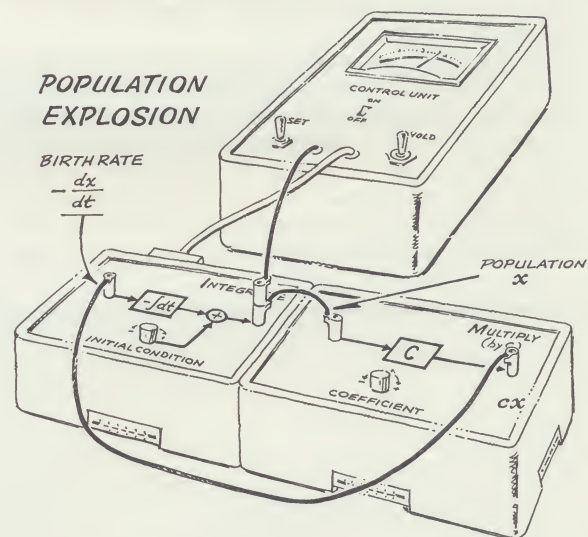
The Population Explosion

Perhaps the most urgent current problem facing the world today is the population explosion. Everyone is familiar with its inexorable exponential behavior. Notice that it involves a loop of cause and effect!

1. Birth rate is proportional to population.
2. Population is the cumulative (integrated) effect of birth rate.

Setting up the PAC, we identify the two variables as *population* and *birthrate*. The two operations are *multiplication* by a constant and *integration*.

PAC Interconnection:



PAC Operation:

1. Set the Initial Condition to the initial population K .
2. Set the Coefficient C to -0.2 as the relation between population and birth rate.
3. Start the problem and plot X values.
4. The result is an exponential curve starting at K .

With the loop of cause and effect graphically illustrated by the PAC units, it is possible to consider the variable as an infinite number of terms, each one relating to the number of times that the signal traverses the loop. Thus, the first trip yields the term Kct , the second $Kc^2t^2/2!$, the third trip $Kc^3t^3/3!$, etc. The answer to the question is not an infinite number of integrators, but one integrator used over and over.

Sinusoids

Oscillatory types of functions occur when a second negative integration operation is inserted into the loop. In the case of population, there might be a negative cumulative effect on the birth rate due to population (such as war). A second Integrator can be inserted into the PAC setup and an oscillatory signal will result.

Mathematically, the signal now goes through two Integrators on each trip around the loop, every other term in the exponential series is removed, and the sign is reversed on the alternate remaining terms.

This is the sine series (or cosine, depending upon where you start).

When inspecting natural loops which oscillate, one should seek two or more cumulative operations. In home heating for example, water must be heated (one integration) which in turn heats the room (a second integration, plus other delays) combine to insure an oscillating temperature.

Studying Causal Loops

After one has studied many causal loops in servo systems, feedback amplifiers, or industrial control systems, one has a good deal of humility about predicting what they will do. The unsophisticated always seem to have one of two reactions when they encounter a loop of cause and effect:

1. It will stabilize. "Supply equals demand"
2. It will explode. "If you raise prices, wages will have to go up, driving prices up, driving wages up, ad infinitum"

In reality, things are rarely so good and seldom so bad.

New Products

Calibrated knobs are now available for all PAC units. These knobs are direct replacements for the present parameter knobs, and permit the user to pre-calibrate his units and set them without having to make a measurement on each setting.

Calibrated knobs were at first avoided on the theory that students should get accustomed to measuring their parameter settings and not learn to trust dial readings. However two PAC applications have been since emerged where calibration would be useful.

The first was suggested by Professor Nathan Cook, at MIT, for generating arbitrary time functions. He used an Adder Unit and Integrator to generate a time function made up of straight line segments of $\frac{1}{4}$ second in length and varying in slope. A problem is run in $\frac{1}{4}$ second segments, changing the Adder setting to prearranged values. The Calibrated dial permits one to set the Adder

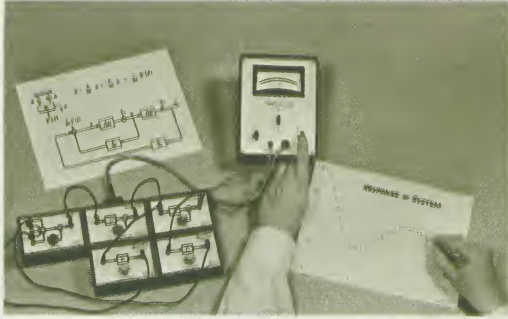
Unit without making a measurement every time.

The second application was suggested by Professor Ludwig Braun at Brooklyn Polytechnical Institute. Graduate Students have been solving Differential equations with time varying coefficients, and non linear differential equations. Solutions to such problems involve a program of coefficient parameter settings with each $\frac{1}{4}$ second increment, and the calibrated dial is a great simplification in this procedure.

Calibrated dials are available from stock in all colors. Orders should indicate number of dials and color desired.

Problem for the next issue.

The next issue will discuss how the derivative is very useful in controlling variables. For homework, how can a log rolling contest be modeled with PAC units? This will be shown in the next issue along with hints on how always to win.



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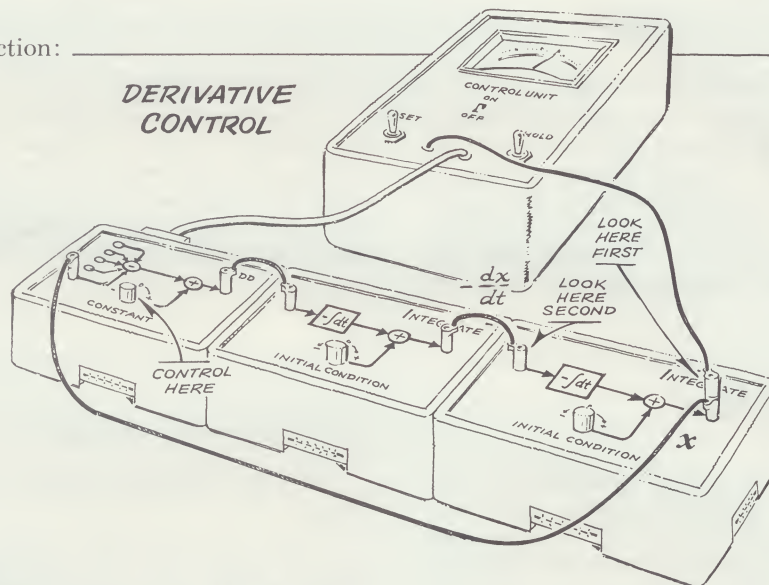
Controlling with the Derivative

Mankind has always tried to control its destiny, but it is only in recent years that we notice the emphasis on the trend, or derivative, of a situation rather than absolute magnitude. We hear of the widening missile-gap, the rising cost-of-living, and the market holding steady . . . all expressions of change or lack of it. Man is becoming increasingly aware of the dynamics of life, and of the importance of controlling the derivative.

Derivative Control

We can illustrate with Personal Analog Computer units the benefits of using derivative control by arranging them as an oscillating system. The PAC operator is asked to attempt to adjust the additive constant in a way which will damp the oscillation and bring the system to rest. His ability to control will quickly improve as he shifts from observations of the variable itself to the derivative, which provides a glimpse into the future.

PAC Interconnection:



PAC Operation:

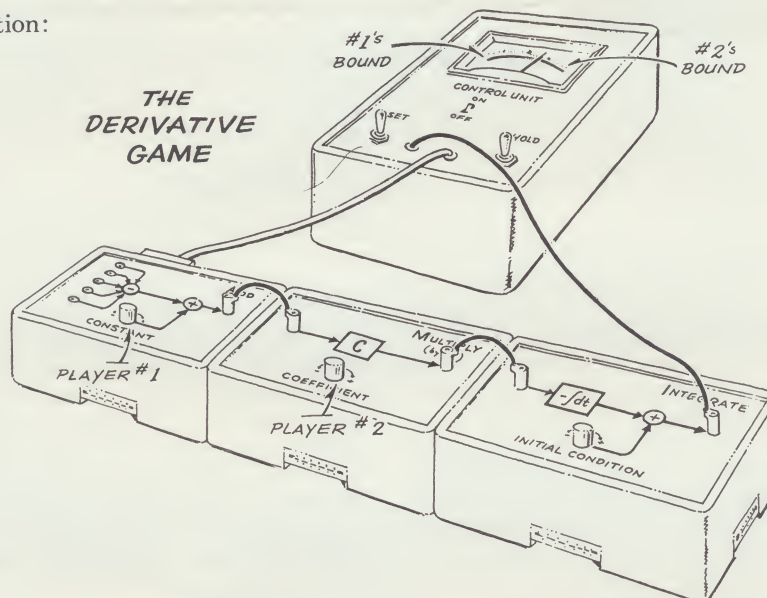
1. Set the Initial Condition to +5 on both Integrators.
2. Connect the meter to the output of the X Integrator, as shown, and start the problem running. Try to make the needle stop moving by turning the Adder Constant knob.
3. Connect the Meter to the output of the X' Integrator and try to stabilize the loop by turning the Adder Constant.
4. Try step 2 again, but this time mentally differentiate the meter motion and respond to the image of the derivative as in step 3.

It is not hard to virtually stop the meter needle motion when you are looking at X'. You need only respond with the control in the same manner as the needle is moving. The only possible way of doing the same thing when looking at X is to mentally differentiate its motion; i.e. when the needle is moving to the left, turn A to a right position and then quickly to the left position as it reverses direction. Since the needle normally moves as a sine wave, you turn A like a cosine wave. If you can stabilize in either situation, it is because you are acting on the derivative signal.

The Derivative Game

To illustrate the benefits of derivative control, we have a game for two people in which the individual who uses derivative control will always win. The object of the game is for the players to drive the needle to their "goal" at either end of the meter scale.

PAC Interconnection:



PAC Operation:

1. Each player takes one side and one parameter knob, either A or C. The two positions are equally good; either player can reverse the move of the other player.
2. Set the Initial Condition to 0.
3. Start the problem situation running.

How to Win Without Actually Cheating

The tendency of a player is to watch the needle, and when it starts to go against him to reverse his knob. This is not fast enough against a skilled opponent. The trick is to pretend to watch the meter, but to secretly look at your adversary's hand. His hand is the derivative of what the needle will do, because of the integrator between the meter and his hand.

A Real-Life Analog

This game is very close to a model of a log-rolling contest. In such a contest, each contender rolls the log back and forth, and if the other fails to reverse his action in time, he is driven to the bound; i.e. water. Log rolling experts always watch each other's feet to the exclusion of all else.

New Products and Improvements

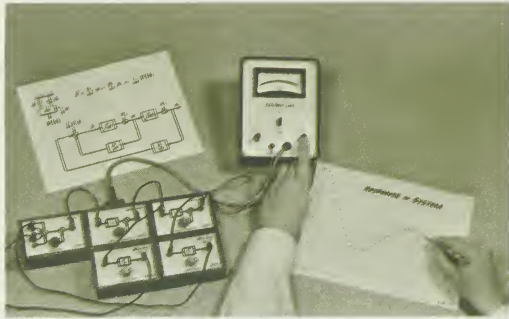
A carrying case is now available for the standard Personal Analog Computer sets. The size of the case is $5\frac{1}{2} \times 11\frac{1}{4} \times 12\frac{1}{2}$ inches, and with the entire complement of PAC units it weighs 7 pounds. The inside of the case has foam-rubber cushioning with compartments for paper, pencils, cables, and the five P A C Units and Control Unit. This case is now included at no extra cost with each purchase of a standard P A C set.

Contained in the Next Issue

Is the Personal Analog Computer non-linear?

Can the important non-linear digital circuits be made, such as the Flipflop, the Schmitt Trigger, or the Multivibrator?

In the next issue, we shall study these questions and others concerning the basic theory and methods of operation underlying binary digital computation.



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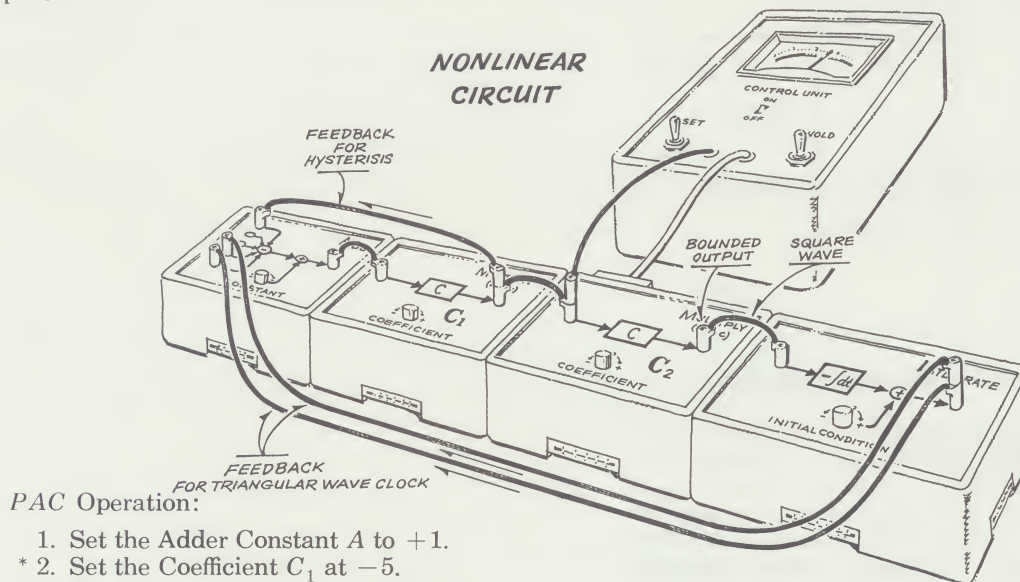
Studying Nonlinear Digital Circuits with the Personal Analog Computer

Are the PAC units nonlinear?

Indeed! They have a very restricted linear range, and then their outputs bound sharply. This is not only a practical necessity but a real advantage, for this same nonlinearity is the basis of digital design. We will illustrate this by setting up a bounded circuit and see how it models digital logic, memory, and timing, the big three fundamentals of the digital computer.

The What and Why of a Digital Computer

The fundamental difference between a digital and an analog computer is the way in which the variable is represented. In the analog units, a single continuous voltage is an analog to the variable. In digital machines, a large number of two-valued voltages represent a single variable through a coding convention. The two-valued voltages are like those



PAC Operation:

1. Set the Adder Constant A to $+1$.
- * 2. Set the Coefficient C_1 at -5 .
- * 3. Set the Coefficient C_2 at $+0.3$.
4. Vary A between limits and notice the C_2 Multiplier output bound. Change bounds by varying C_2 . The bounds are normally beyond the meter range, so the second Coefficient unit with gain of less than 1 brings the bounded variable back into range.
5. Add the connection from C_1 to the input of the Adder to make a hysteretic loop. Vary A and note the snap action.
6. Add the Integrator and feed back its output to the input of the Adder to get a free-running multivibrator action.

* Set C prior to interconnecting units.

in Operation 5 of the experiment above, bounded either at one limit or at the other. The resolution of the variable possible with n two-valued voltages is equal to 2^n .

Why is this done? A modern flip-flop circuit such as is modeled requires almost the same cost as the PAC units involved, and yet 10 of them would be needed to represent a voltage as well as a single PAC unit can. The answer is that the binary variable can be operated on, stored, and switched any number of times without degeneration. A variable can be withdrawn from a storage, switched to logical circuits, operated on mathematically, and finally re-

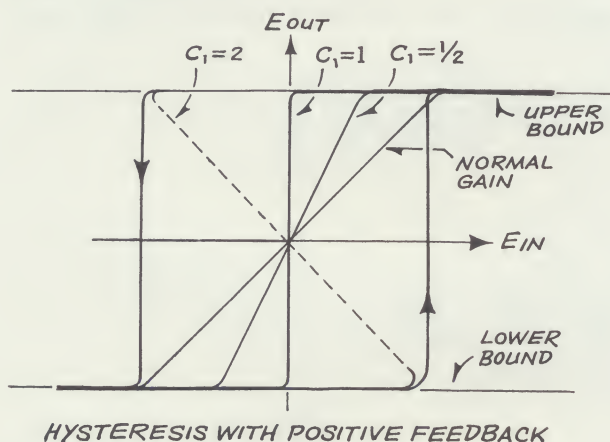
turned to storage as frequently as is necessary without degrading its accuracy in the least! Such is not the case with analog computers, so they are completely unsuitable for many operations — accounting for example.

Digital Computer Components

Most of the interesting components of digital computers are used for logic, memory, or timing.

Logic is illustrated with the simple bounding circuit arrived at in Operation 3 above. Logic means that an output condition exists, if certain input conditions are met. Only the two bounded conditions are assumed to be possible. Because of the high gain and the bound, if a signal is inserted into any of the Adder inputs other than zero volts, a signal will appear at the output of C_2 . This is a logical circuit, and all adders, multipliers, comparators, etc., are extensions of this basic device.

Memory becomes apparent when you have a device which retains the effect of an input after the input is removed. Putty, paper with pencil marks, and the brain are common examples. Electronically, memory means that a voltage signal continues to come out after you have put it in. A practical memory must be permanent and also erasible. It should have the following hysteresis curve:



We can get memory in the arrangement of PAC units shown by adding positive feedback as in Operation 5. This forms a positive loop of gain greater than 1, which will either go to one bound or the other. Move A to the right or to the left and then turn it back to zero. The meter output indication will retain the memory of where you had A last.

Figure 2 shows how the hysteresis curve results from increasing positive feedback. When the total loop gain is exactly 1, the element has infinite gain, but no hysteresis. At gains of greater than 1, the loop opens up as indicated.

The hysteresis loop is the basis of magnetic memory. The same loop is used in flip-flop design.

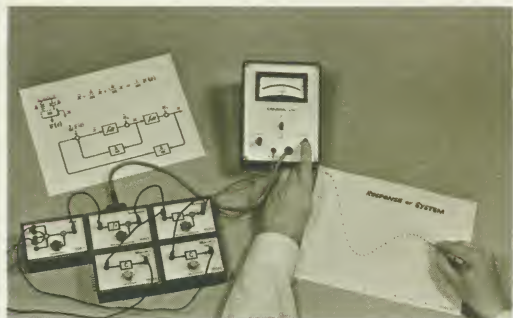
The great virtue of a digital computer is that it can do a sequence of operations, so that although an Adder or Multiplier is very expensive, it can be used for many different parts of the problem in a sequenced program. This sequencing implies a series

of timed markers to control the order of the switchings and operations. Addition of an Integrator to the hysteresis circuit just discussed, as it was done in Operation 6, will produce such a clock. Varying C_2 will change the pulse rate, and changing A will affect the duty cycle. At the output of the Integrator is a triangle wave, which is frequently used in electronic test equipment.

It is pleasant to speculate that digital computers are made up of analog modules after all. At the philosophical level, the differences between analog and digital are largely superficial. The real difference is the representing of a variable by a continuous voltage or a numerical incremental voltage. It is a fundamental one and usually determines which type of computer is appropriate for a given job.

In the next issue

The next issue will discuss the basic elements of operational calculus, where it ties in with "frequency response," and how it can be demonstrated and studied with the Personal Analog Computer.



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"Frequency is not necessarily zero crossings per second,
but more generally what a function is doing with respect to itself"

Why Transfer Functions?

Given a complicated system such as a nuclear power plant or a large machine or an operating set of the *Personal Analog Computer* units, insert a control signal, and what comes out? What is the result?

The relationship between a system's response and its input stimulus is called the "Transfer Function". Many engineers make their living studying and predicting transfer functions using such lore as frequency analysis, natural frequencies, superposition, complex algebra, unit impulses, and operational calculus. These subjects are the basic tools for relating outputs of systems to their inputs.

A Personal Investigation into Transfer Functions

At our disposal in the form of the *Personal Analog Computer* is a malleable system which can be easily modified and measured under differing conditions. Starting from the beginning, let us work out our own methods for dealing with transfer functions using the P A C as an aid.

We start with one Integrator as the simplest, basic, non-trivial, dynamic element.

The first experiment involves measuring the output of the Integrator with nothing going in. The output can be set to various values, which it sustains without an input signal, and my conclusion is that the transfer function is apparently infinite for the test condition.

The second experiment is to put something into the Integrator from an Adder. Using a constant input signal, the output of the Integrator is not constant. If it is plotted, as in Figure 1a, I conclude that the output is dependent on how long the input is applied.

As a third experiment to determine the output dependence on the input amplitude, I apply several signals of different amplitudes for the same time duration. I observe that the output response is directly proportional to the amplitude of the input signal.

Now, I try to summarize my observations. It appears that the output depends equally and linearly on how long an input signal is applied and how large it is. The Integrator seems to be linear: twice the input amplitude, twice the output result; and it seems to be continuous: twice the time of an input signal, twice the response. A reasonable cause and effect description would be that it is a device which continuously makes measurements of its input and accumulates the measurements at the output. The Integrator weights each infinitesimal of input signal equally regardless of when it occurred.

A device which merges an infinite number of samples of a signal, treating them all equally, and summing them for an output signal well deserves to be called an "Integrator".

A More Complex System

In Figure 2 we have a more complicated array of P A C units than the single Integrator. In anticipation of the probability that both duration and amplitude will influence the output, let us insert the simplest test signal that we can imagine: a fixed value of 10 analog units for $\frac{1}{4}$ second, choosing the highest amplitude and the shortest measureable time with the standard P A C units. We call this pulse the "Unit Impulse" and use the Adder to generate it. The Adder Constant is set to 10 analog units for the first $\frac{1}{4}$ second of the run of a problem, and then it is reset to zero for the rest of the run. (This is only an approximation to a unit impulse.)

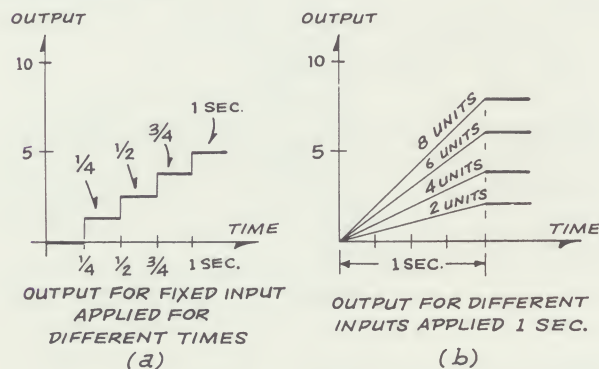


Figure 1

P A C Interconnection:

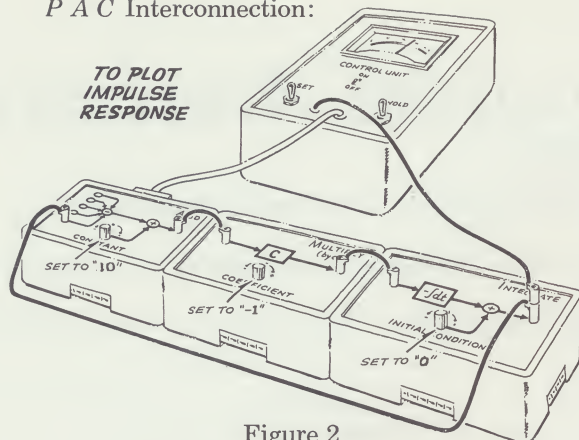


Figure 2

P A C Operation:

1. Remove the input from Adder
2. Set Adder Constant to +10, Coefficient to -1, and Integrator Initial Condition to 0
3. Reconnect Adder Input and run problem for $\frac{1}{4}$ second
4. Reset the Adder Constant to 0, run for successive $\frac{1}{4}$ -second periods, and plot

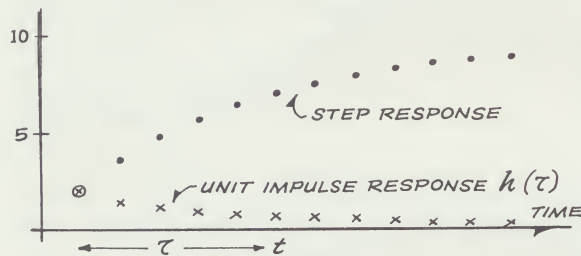


Figure 3

The plot of the output of the P A C system is shown in Figure 3, and it is called the "Unit Impulse Response". It really tells the whole story about the system dynamic response. To show this, try another simple input function such as a step, and plot the output response as in Figure 3. The step response is the sum of the impulse responses initiated every $\frac{1}{4}$ second. This is obvious because our method of applying the step is merely to put together a series of $\frac{1}{4}$ -second impulses. Therefore, it is entirely logical that the output should be a series of $\frac{1}{4}$ -second unit impulse responses added together!

The significant part about the discovery of the unit impulse response is that I can make any input function (besides the step) out of a series of $\frac{1}{4}$ -second impulses by adjusting the amplitude every $\frac{1}{4}$ second in order to approximate the desired input function. Therefore, I can predict the system response to any input as the weighted sum of a delayed sequence of impulse responses.

Now, I can apply the same method to any system even without knowing what it contains. I apply the unit impulse to the system and plot its output. Then, for any input function, I superimpose the unit impulse responses starting them at the proper times and in the proper proportions.

$$f_o(t) = f_{in}(t)h(0) + f_{in}(t-1)h(1) + \dots + f_{in}(t-\tau)h(\tau)$$

$$f_o(t) = \sum f_{in}(t-\tau)h(\tau) \rightarrow \int f_{in}(t-\tau)h(\tau)d\tau$$

Unit Impulse Response for $t=0$ Unit Impulse Response for $t=1$ sec.

What input is now What input was 1 sec. ago

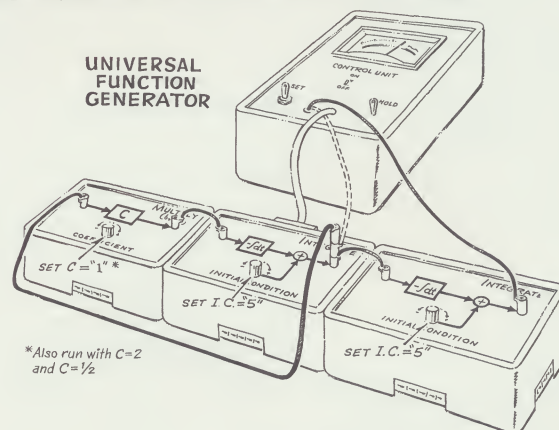
What output is now What input was τ seconds ago What output would be τ seconds after impulse

The method of predicting the output of any system caused by any input function is called "Superposition",* since it involves the overlaying of the unit impulse responses. Appreciating the fact that it is approximate because of the $\frac{1}{4}$ -second time increments which I was forced to use with the P A C units to piece together the input function, I assume that the time interval could be made smaller if necessary to give more accurate results. Of course, with a reduction in the time interval, the system response would decrease unless the amplitude is increased. In the limit, then, the unit impulse consists of a pulse which is very narrow but very high and has a unit area.

Frequency Analysis

Unfortunately, as practicing engineers our enthusiasm for superposition is shortlived. Even if we master analytical techniques of evaluating integrals, so that we can avoid many $\frac{1}{4}$ -second measurements, we find that it is just too much work. After becoming bogged down on complicated integrations, we ask ourselves if we should not specialize in the prediction of the responses of systems to certain simple input functions such as the step.

Pursuing this possibility, we ask a logical question: "Is there any function which can go through a system, such as an Integrator, and emerge unchanged except for a scale factor and possibly an additive constant?" If we could find such a function that would go through one Integrator unchanged, then it would go through any number of Integrators unchanged as well as combinations of Integrators, Coefficient Multipliers, and Adders with only scale changes and additive constants! If such a function could be discovered, and if people would use this function or combinations of this function as system inputs, then we would be back in business.

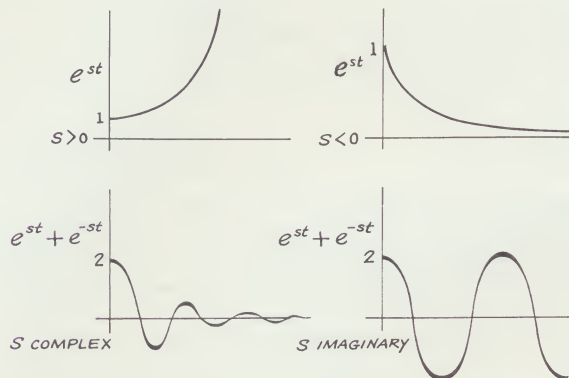


*Also run with $C=2$ and $C=\frac{1}{2}$

Figure 4

* See a book on Electrical Engineering such as "Electronic Circuits, Signals, and Systems" by Mason and Zimmerman, page 320.

The way to find the universal function is to connect the output of an Integrator to its input, in which case we have forced the output to equal the input, and to observe the result. If a response function results, it must be the function which we seek. Such a function does exist, namely e^{st} , where s is proportional to the Coefficient Multiplier setting C . We find that the signal goes through an Integrator in the same form, but that it is multiplied by $1/s$. Thus, we can treat an Integrator merely as a Coefficient Multiplier with a factor of $1/s$. Any combination of Personal Analog Computer units will respond accordingly.



SOME TIME FUNCTIONS OF THE FORM e^{st}

Figure 5

Figure 5 shows some of the many forms that e^{st} can take, if we allow s to take on complex values. It is plausible to think that most time functions can be made of functions of e^{st} , and people have worked out tabulations showing the conversion of common time functions to this form just for the purpose of predicting the response of their systems.

Natural Frequencies

The quantity " s " in the discussion above is called frequency and can have positive, negative, and complex values.

Occasionally a system, such as the Integrator, puts out a signal although there is no input, which means that it has infinite gain. This output signal has a certain definite frequency which is called the resonant or natural frequency. To find this frequency, you compute the transfer function of the system substituting $1/s$ for each Integrator. For example, the lag in Figure 2 has a transfer function:

$$\frac{C}{C + s}$$

Setting the denominator equal to zero gives you the condition where the transfer function is infinite. The root or roots of the denominator are therefore the natural or resonant frequencies. The term natural frequency is a good one, because it indicates the ability of the system to generate such frequencies without their being present at the input.

New Product Announcement

Now available is a new *Personal Analog Computer* unit: The *Precise Signal Source or Coefficient Multiplier*. It can provide a fixed or manually-variable voltage as a precision source of signal for function generation. Any arbitrary function can be generated by varying the *Precise Signal Source* control settings through a sequence of amplitudes representing the input function. It can also serve as a pre-calibrated *Coefficient Multiplier*.

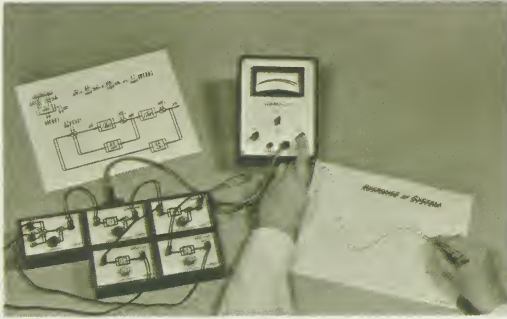


Price Reduction

Again, a reminder that *PAC* prices have been reduced due to design and production economies. Also, experience with sets in use has been so favorable that *PAC* units are guaranteed absolutely against malfunction.

Discussed in the next issue

Diversity of application of the *Personal Analog Computer* is shown by the solution of a fundamental arithmetic problem, which is to find the square root of two (an impossible problem for a digital computer). We have received four distinct solutions: one from a Freshman calculus student, a digital computer programmer, a practicing electrical engineer, and from an analog computer expert.



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Finding the Square Root of Two

The four *Personal Analog Computer* solutions shown here to the problem of finding the square root of two do not exhaust the possibilities, but they do illustrate widely different viewpoints and approaches to the problem.

Taking a root is the inverse of raising a number to a power. Historically, when men have tried inverse operations, they have always discovered new types of numbers to which they then gave complimentary names such as negative, fractional, imaginary, or irrational. When taking the inverse of addition (subtraction), negative numbers were discovered. With the inverse of multiplication (division), fractions appeared, and with the inverse of raising to a power (rooting), the imaginary roots of negative numbers were discovered. Some numbers were found which were not the ratio of integers, and these were called irrational numbers.

The square root of two was the first irrational number to be discovered by Pythagoras. He showed that it was different from other numbers, because it could not be expressed by a ratio of integers or by the equivalent to a decimal of finite length. After the mathematicians overcame their initial revulsion toward irrational numbers, they went on to discover many important and useful ones including π and e .

Freshman's Solution

The Freshman's solution is the most brute force of them all. However, he has just had calculus, to him it seems like a useful new tool, and he is proud of using it. He simply notes the fact that the second integral of A is $At^2/2$ and proceeds to set up two Integrators as in Figure 1.

For convenience, he selects a value for A of $+2$, so that $A/2$ is $+1$. Starting both Integrators from Initial Condition equal to zero, he lets them run until t^2 at the output of the second Integrator equals $+2$. Then, he can get the square root of 2 by halving the output of the first Integrator, which is:

$$At = +2t = +2\sqrt{2}$$

if $t^2 = +2$.

Digital Computer Programmer's Solution

The next solution to our problem proposed by a digital computer programmer is unashamedly approximate, because a digital computer by definition can not generate an irrational number.

P A C Interconnection:

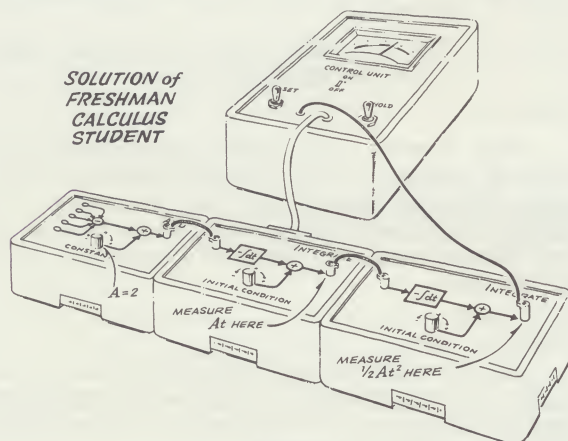


Figure 1

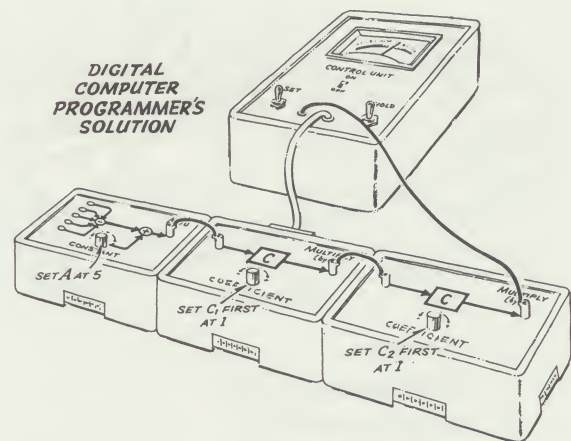


Figure 2

P A C Operation:

1. Set the two Coefficients C_1 and C_2 to exactly +1.
2. Set the Adder Constant to +5.
3. Have the Meter indicate the output of the second Multiplier, and it should read +5 if everything has been properly adjusted.
4. Now, the idea is to move C_1 and C_2 by the same amount so that the Meter reads +10. This is done by changing C_1 until the Meter reads slightly higher (+5.5). Then change C_2 until the Meter goes the same amount

higher than the first setting (+6.0). Now, you have actually moved C_1 more than C_2 , because you were multiplying a larger number by C_2 to get the same change at the output as when multiplying C_1 by a smaller number. However, if you make the changes very small, then the error is not large. Also, you will get further compensation if on the next increase of the Coefficients C_1 and C_2 you again change C_2 before adjusting C_1 . This process is continued until the Meter reads +10 as is shown numerically in the tabulation:

A	5	5	5	5	5	5	5	5	5	5	5
C_1	1	1.10*	1.10	1.10	1.18*	1.27*	1.27	1.27	1.34*	1.42*	1.42
AC_1	5	5.50	5.50	5.50	5.90	6.35	6.35	6.35	6.70	7.10	7.10
C_2	1	1	1.09*	1.18*	1.18	1.18	1.26*	1.34*	1.34	1.34	1.41*
AC_1C_2	5	5.5	6	6.5	7	7.5	8	8.5	9	9.5	10

* indicates the Coefficient which has been changed in each step

The two Coefficients C_1 and C_2 will then be very nearly equal and have gains of $\sqrt{2}$, since $C_1C_2 = 2$ and we have made $C_1 = C_2$.

Electrical Engineer's Solution

Every well-informed engineer looks on electrical signals as vectors which are rotating at frequencies ω and perhaps also varying in length. In particular, he interprets the signal in the oscillator in Figure 3 as a single rotating vector of fixed amplitude, and the signal at the output of one Integrator as the projection of that vector on the X axis, while the signal at the output of the other Integrator is the projection on the Y axis. If he sets an initial condition of +B on each Integrator, he is creating a vector at 45° , and its length is $B\sqrt{2}$. He can read this length on the Meter by running the problem until the vector reaches the X axis. Actually, he can look at the output of either Integrator and measure the peak value of the sine wave which appears at the outputs.

P A C Interconnection:

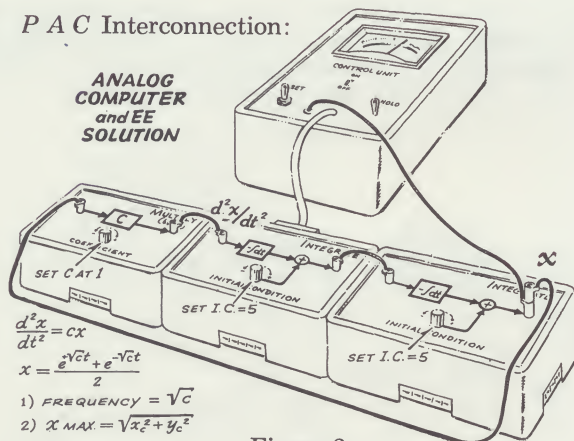


Figure 3

P A C Operation:

1. Set Integrator #1 to 5.
2. Set Integrator #2 to 5.
3. Set $C = 1$.

4. Run and plot output of #1 and #2 Integrators.
5. Repeat with $C = 2$ and $C = 1/2$.

Analog Computer Operator's Solution

Reviewing the equations governing the P A C system of Figure 3, we remember that the solution is:

$$x = \frac{e^{+\sqrt{C}t} + e^{-\sqrt{C}t}}{2}$$

When C is negative, as it is here, this is a sine or cosine wave of frequency $\sqrt{C}$.

This suggests measuring frequency at two values of C knowing that the ratio of the frequencies will be in the ratio of $\sqrt{C}$.

P A C Operation:

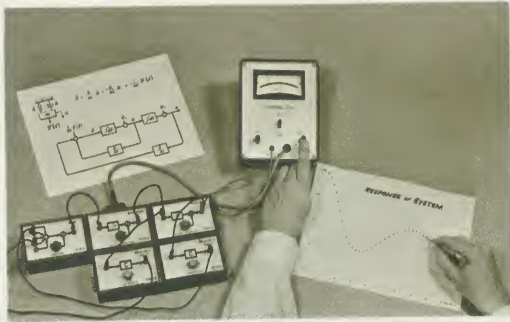
1. Make Coefficient $C = +1$ and measure the frequency of oscillation.
2. Increase C to +2 and re-measure the frequency.
3. Take the ratio of the frequencies, which will be the square root of 2.

Conclusion

This exercise tied together such diverse concepts as numbers, calculus, vectors, iterative operations, electrical engineering, and computing. It is intended as an exercise for the student, because he must use knowledge from several disciplines to solve his problem.

Advance Notice

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Feedback and the Operational Amplifier

"Feedback is Salvation" G. A. Philbrick

Although the concept of feedback has been applied in diverse situations for centuries, within the last 50 years a great impetus was given to the analysis of feedback first with the introduction of mechanical servomechanisms and later as electronic communication and control became more important. To the novice, the process of feeding some of the output of an amplifier back to the input seems like an insane procedure likely to lead to an endless spiral of increasing signal; or conversely, he senses intuitively the possibly beneficial effects of feedback. As a result, thousands of engineers make a good living today using feedback in the solution of their problems; and hundreds more make a better living helping people to solve the problems which feedback creates.

In principle, feedback involves measuring a variable which one is trying to control, comparing its value with the desired value, amplifying the difference, and applying the amplified "error signal" to some control point in the system which will affect the output variable in the proper manner. The resultant affect on the output is normally expected to be a modification so that the difference becomes zero.

The prime virtue of feedback is that you do not have to know the exact relationship between the manipulating point and the variable under control, as long as you can make a precise measurement of the output variable and change the control point in the right direction. For example, you might not be able to predict the amount of fuel needed to heat a furnace to 800 degrees for one hour, but you can certainly add fuel until the temperature is approached and then cut back and hold at that temperature.

When the general features of feedback become familiar, we discover that it is almost universal in nature. We find it in complex industrial managerial and social situations. We find it in the manual skills of the individual, gauging his work with his eye and manipulating with his hand. We even find it on the microscopic level within the chemical and nervous processes in the human body.

Feedback and System Synthesis

Systems with feedback are subject to analysis, and an entire study of linear and non-linear effects in systems is possible without venturing into the laboratory. Conditions of stability and system response to a variety of inputs are the common objectives of such analysis. As will be shown, an analytical study used as the feedback function in a high-gain design loop causes its opposite of system synthesis, which is usually the ultimate objective, to be the forward-acting function.

A simple illustration of feedback principles can be made using a *Personal Analog Computer Adder and Multiplier*. The input variable to the *P A C* setup is either an electrical signal at the Adder input or a movement of the Adder Constant.

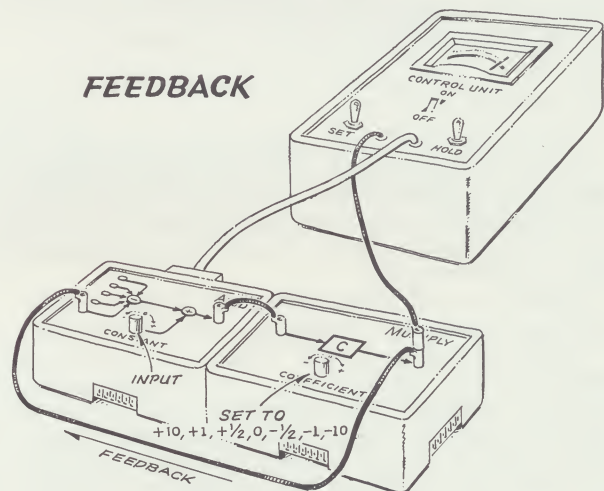


Figure 1

Typically, as in servomechanisms and high-fidelity amplifiers, feedback is negative (degenerative), and this is the result observed if the Coefficient *C* is positive.

Positive feedback (regeneration) is not necessarily destructive to a linear input-output characteristic, as long as the loop gain is less than +1. This can be seen by making *C* have a value between 0 and -1.

When C is made -1 (still positive feedback because of the signal polarity reversal in the Adder), any input will cause a full-scale output.

What happens when the loop gain is more positive than 1 ? Actually, the same input-output relationships still hold, but the sudden appearance of hysteresis obscures their operation. The hysteresis is merely a manifestation of the presence of bounds in the unstable system.

Stability

People are concerned with two types of instability. One occurs when the loop becomes "hung up", i.e. when it goes to an extreme state and will not come out of it. Obviously, this is due to a loop gain greater than $+1$. We see the second condition of instability when the loop signal varies periodically despite the fact that nothing is going into the loop. This means that there is some signal which can go around the loop without being altered, i.e. a frequency for which the gain of the loop is $+1$. This applies to all instabilities including non-linear ones.

The majority of analysis is done considering sine-wave signals. By the more general criterion above, if we can get a loop to oscillate at some frequency, then we know that the loop gain for this frequency must be exactly $+1$. The question now should be: How can this happen, if we made the d-c loop gain negative? The answer is that the gains for different frequencies are different.

Study an Integrator. Its gain is inversely proportional to the frequency, as far as absolute magnitude is concerned, and an input sine wave is shifted in phase by 90 degrees. Two Integrators in series will shift a sine wave by 180 degrees; therefore, they act on a sine wave as an inverter (with gain or attenuation depending on the frequency). If two Integrators are put in a negative loop, gain can then be $+1$ for at least one frequency.

Simulation of Systems

Venturing, now, from analysis into the realm of the experimental study of systems, what techniques are available? The engineer can build the system from calculations and then proceed to modify it to meet the desired objectives. This is often prohibitive from a cost standpoint.

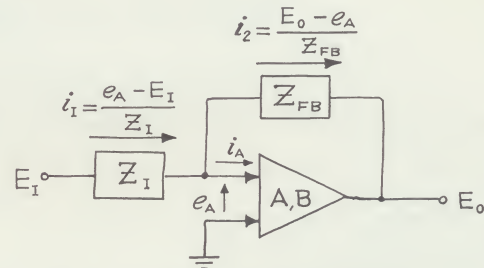
The use of pilot plants and scale models is also a traditional approach to obtaining experimental data on which to base a design. Usually, the tests are limited to the static or steady-state performance as is the case with wind tunnels and ship towing tanks. Dynamic response from models is virtually impossible to obtain, because of the difficulties of instrumentation, limited bounds, fixed conditions, and often the impossibility of scaling time as an independent parameter so that results are obtained within a useful time scale.

Concurrent with the need for prediction of the dynamic performance of systems, methods were developed for the study of electronic circuits, which are almost always in a state of change. It became apparent that the same mathematics related to the dynamic response of electronic and mechanical systems. Having the common element, it was then logical to use electronic circuits as simulators for non-electronic processes.

Function Synthesis and the Operational Amplifier

An engineer may want to synthesize functions for a number of excellent reasons. He may wish to simulate a system for study and analysis, before it is built; he may wish to have a data-processing link; or he may wish to design a particular signal modifier as part of a real-time regulator.

Before building system models, one must be able to make unit transfer functions such as Z_{FB}/Z_I in Figure 2. It is possible to build a variety of complex systems using only passive elements. However, the method is awkward and limited, and the logical result is to add active elements, namely amplifiers, to reduce loading effects and signal attenuation and to provide power gain.



A IS AMPLIFIER VOLTAGE GAIN
 B IS AMPLIFIER CURRENT GAIN

$$E_0 = -Ae_A \quad \text{AND} \quad i_2 = -Bi_A$$

$$i_I = i_2 + i_A$$

$$\frac{e_A - E_I}{Z_I} = \frac{E_0 - e_A}{Z_{FB}} + i_A$$

LET A AND $B \rightarrow \infty$, THEN e_A AND $i_A \rightarrow 0$

$$\frac{-E_I}{Z_I} = \frac{E_0}{Z_{FB}}$$

$$\text{GAIN} \frac{E_0}{E_I} = -\frac{Z_{FB}}{Z_I}$$

Figure 2

By using operational amplifiers, we have the following important advantages:

1. The operational amplifier has negligible output impedance and a known, fixed input impedance. Loading troubles are minimized.
2. The signal flow is in one direction only, so that isolation between functions is achieved, and system synthesis is much simplified conceptually.
3. When an element is used in the feedback path, its inverse is formed as far as signal flow forward is concerned. Thus, a capacitance will behave like a feed-forward inductance, when the capacitance is put in the feedback path. This means that the equivalent of RLC networks can be built without using inductors, and one can even have inductor characteristics that could never be built.
4. The transfer functions which are synthesized depend only upon the performance of the passive elements, since the active operational amplifier works to eliminate the affect of its own characteristics by having a very high gain.

5. Almost every basic electronic circuit including flip flops, multivibrators, oscillators, filters, amplitude selectors, comparators, timers, sawtooth generators, and many others can be built to accurate specifications using the operational amplifier as the central component.

One might expect that the goal in the operational amplifier design would be to have some modest gain such as 1, 2, 10, 100, or 1000, but it turns out that the ideal operational amplifier has:

1. Input impedance high
2. Voltage and current gains very high
3. Output impedance low

How the Operational Amplifier Works

Beneath Figure 2, the mathematics of the operational amplifier shows that the overall transfer function is the ratio of feedback to input impedances. The behavior is easy to understand in terms of feedback. The amplifier input voltage and current must be zero for the output voltage to be finite or reasonable, since the output drives the amplifier input back toward zero via the feedback impedance. One way to test for proper operation of an operational amplifier is to measure the voltage at the input point.

The overall transfer function is defined by Kirchhoff's Laws of voltage and current summation. The current through the input impedance is calculated as the input voltage divided by the input impedance, and this is the same as the current going through the feedback impedance (since there is nowhere else for the current to go). Hence, the output voltage is determined as the ratio of the feedback to input impedances times the input voltage.

Differentiator Synthesis

As an example of the application of an operational amplifier, we will design a differentiator using *Personal Analog Computer* units. First, an operational amplifier is made from an Adder and Multiplier setting the Coefficient C to -1 , which makes the loop gain of the combination infinite. Then, we put a PAC integrator in the feedback path, and the result is a feedforward function which is the inverse of integration, i.e. differentiation. Change the Adder Constant and note the output of the Multiplier.

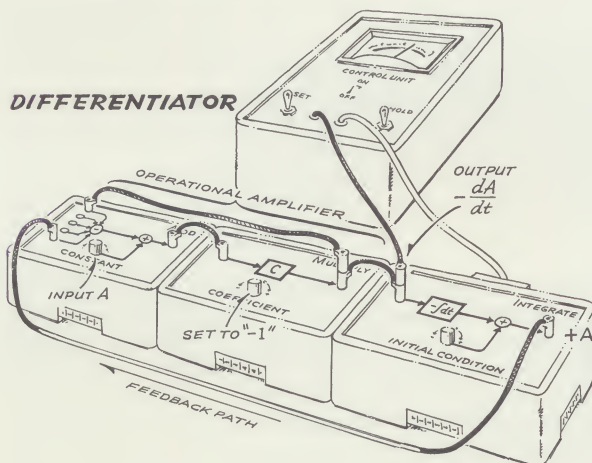


Figure 3

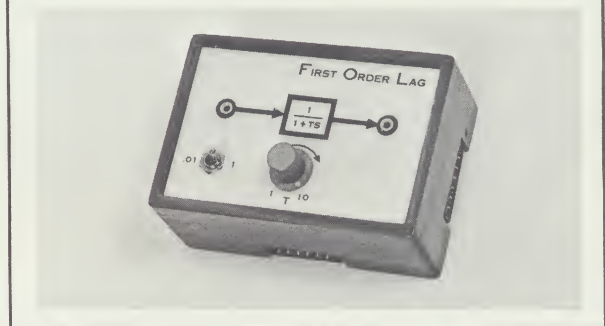
The Future of the Operational Amplifier

At the present time, the operational amplifier is the heart of the analog computer, and it is being used increasingly in instruments as well. Engineers discover that when they design sawtooth circuits, linear amplifiers, adders, averaging circuits, they are really redesigning the operational amplifier. They are turning to this universal package which will provide an accurately specified performance with appropriate passive elements.

For example, *Personal Analog Computer* units use operational amplifiers as the least expensive and most accurate method of getting the operations of summing, integration, and linear amplification. In the future, with the introduction of microcircuits, the operational amplifier may well replace the transistor as a general-purpose active component.

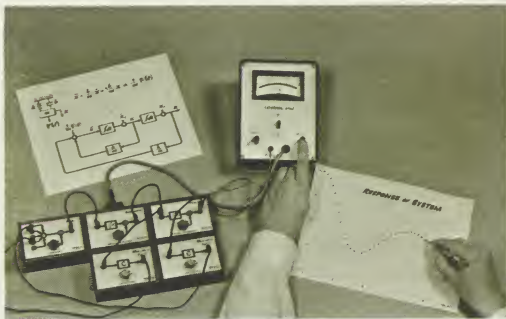
New Products

An important linear dynamic simulation function is the First Order Lag, which we indicated in Figure 2 of Issue No. 4 could be made with 3 *PAC* units: an Adder, Multiplier, and Integrator. Now available as a single *PAC* unit is the non-inverting First Order Lag, which has a lag time constant adjustable between 0.1 and 10 seconds.



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Solving Non-Linear Differential Equations

An engineering problem involving non-linear differential equations is an excellent experience for the student. Non-linear problems are far more common in nature than linear and are the primary justification for using computers. Even if experiments with non-linear problems only confuse the student, they will make him appreciate more the concept of linearity and the reason behind its simplifying approximations.

It is good to be able to introduce the students to non-linear differential equations before he becomes too fixed in his ways. Linear theory is sophisticated and charming, but it often proves to be a superficial trifle when it comes to worldly matters such as solving real problems.

The problem we have chosen for this issue is one which is extremely simple; however, it illustrates some important aspects of systems in general and non-linear systems in particular. It also shows the student that certain intuitive ideas do not always work out as expected.

One interesting facet of the problem presented this month is the presentation of the alternatives of buying the additional non-linear equipment offered or using ingenuity to solve a non-linear differential equation with standard PAC equipment, which is extremely linear (within its operating range). The choice between hard cash and hard work is not one which is usually presented in schools, and perhaps it is the most important part of this lesson.

The Can Problem

A cylindrical can is full of water, and it has a little hole in the bottom. Because of the hole and gravity, water is coming out, the level in the can is going down, and generally speaking we have a dynamic situation.

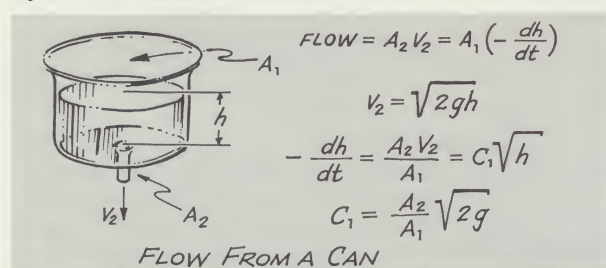


Figure 1

The first step is to identify the variables. They are the height of the water in the can and the flow of the fluid out of the bottom.

Next we must relate these variables by operations. The volume of water in a can is related to flow by linear integration. Whether the height also is a linear integral of flow depends upon the shape of the can; it is if the horizontal cross section is independent of the height. For example, if the can were conical, we have an integral volume relationship, but the rate of change of height is greatest toward the small end of the cone. A non-linear element is needed to convert the volume into height.

Another non-linear relationship enters the problem if the flow rate also depends upon the height of water, as it does when the can is emptying through a hole in the bottom. As height decreases, flow also decreases. However, it is not linear, but is proportional to the square root of height. One viewpoint of the velocity-height relationship comes from energy considerations. A small volume of water at height h disappears, and its potential energy appears as kinetic energy of water squirting out the bottom with a velocity V_2 .

Consider each particle of water leaving the can as having fallen from the top of the water level. In free fall it loses potential energy of mgh and gains kinetic energy of $\frac{1}{2}mV_2^2$. If one particle has collisions with other pieces of water, it transmits its energy to them, and they either come out with velocity V_2 or pass the energy along to particles which do. Equating kinetic and potential energies:

$$\frac{1}{2} m V_2^2 = m g h$$

so that:

$$V_2 = \sqrt{2 g h}$$

The derivation assumes that a particle of water on the surface has no initial velocity, which is almost true if the hole in the can is small.

We can immediately make a block diagram (Figure 2) showing these two operations and choose PAC units to model this system. Unfortunately, it involves a square rooter — not contained in a standard PAC set. Now we must decide whether to buy or work and think.

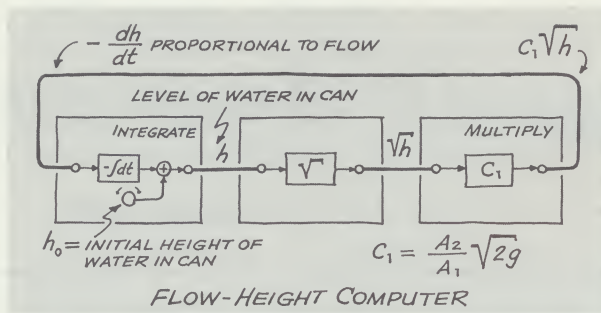


Figure 2

A Square Root Computer

The most common result of the above situation is that the engineer expends some effort and thinks first, but then he buys if the same problem arises often enough. We hope that the student will do the same here. The May issue of *The Dynamic Analyzer* showed 4 different square-root computers. We suggest here the square-root computer consisting of two Integrators in cascade as in Figure 3 below:

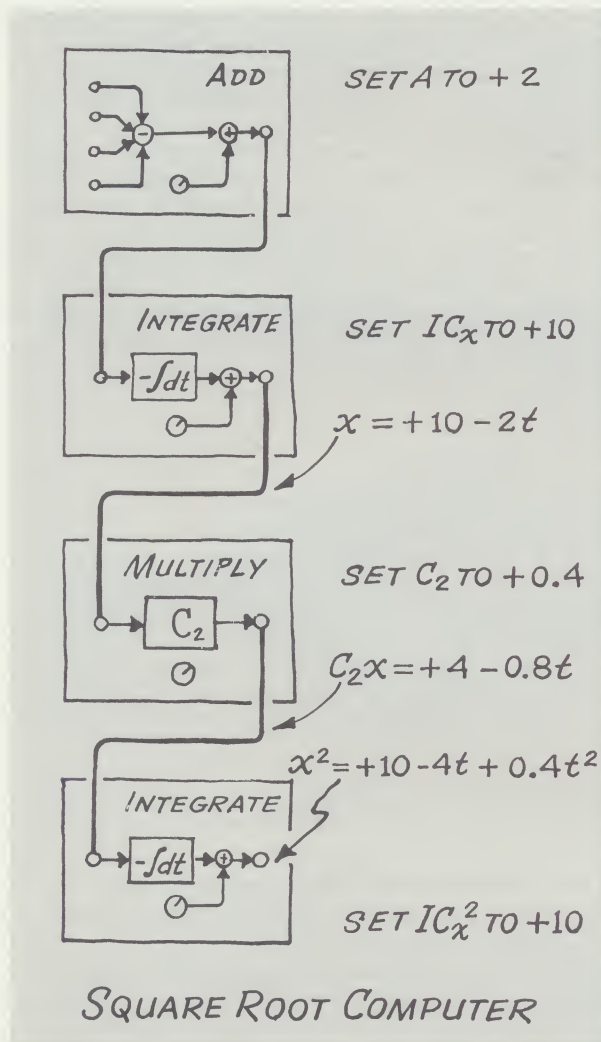


Figure 3

The principle behind this method is that successive integrations of a constant yield a function $x(t)$ which changes linearly with time t and a second function $x(t^2)$ which varies with t^2 . We can run these Integrators until $x(t^2)$ has the desired value (which may be h in the can problem) and feed its square root $x(t)$ (which would be $\sqrt{h}$) into the problem. This computer will require a second PAC set, which is operated separately with only the ground connection common to the first.

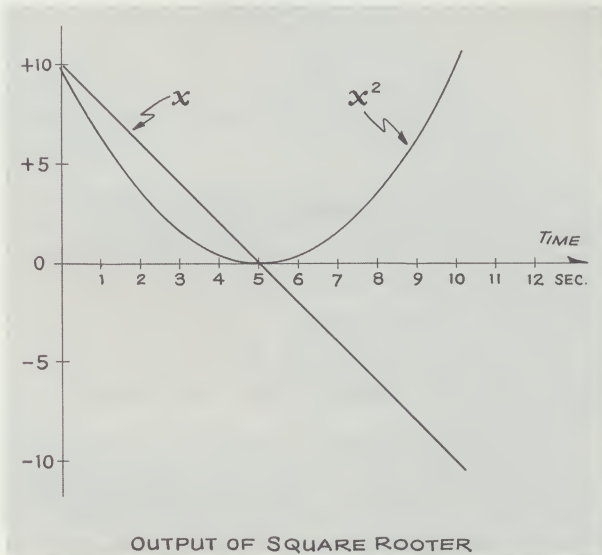


Figure 4

In Figure 4 we have plotted how the square root computer reads out with time. We calibrate it by setting the Constant into the first Integrator to $+2$, so that the first Integrator output changes slowly. Set the Initial Condition of both Integrators to $+10$. Set Coefficient to about $+0.4$. Now run the system continuously and observe if X^2 behaves properly, i.e. goes down exactly to 0 and then rises again. If it goes negative, increase C . If it doesn't reach 0, decrease C . If it exactly reaches 0 and reverses, you are assured that the two Integrators have values that are exactly related by a square root relationship.

Full Scale of both functions represent 1, since:

$$x(t) = \frac{x}{10} = +1 - 0.2t$$

and

$$x(t^2) = \frac{x^2}{10} = +1 - 0.4t + 0.04t^2$$

so that

$$x(t^2) = (x(t))^2$$

Some Sample Solutions

In Figure 5 we have the entire Single Can PAC computer. With Meter #1, read h ; and with Meter #2, read x^2 from the square root computer. We will assume that the constants associated with the can analog add up to 1. Then the height Integrator is fed by $C_1 \sqrt{h} = -dh/dt$ from the square-root computer, and x^2 is made to track the Integrator's output h .

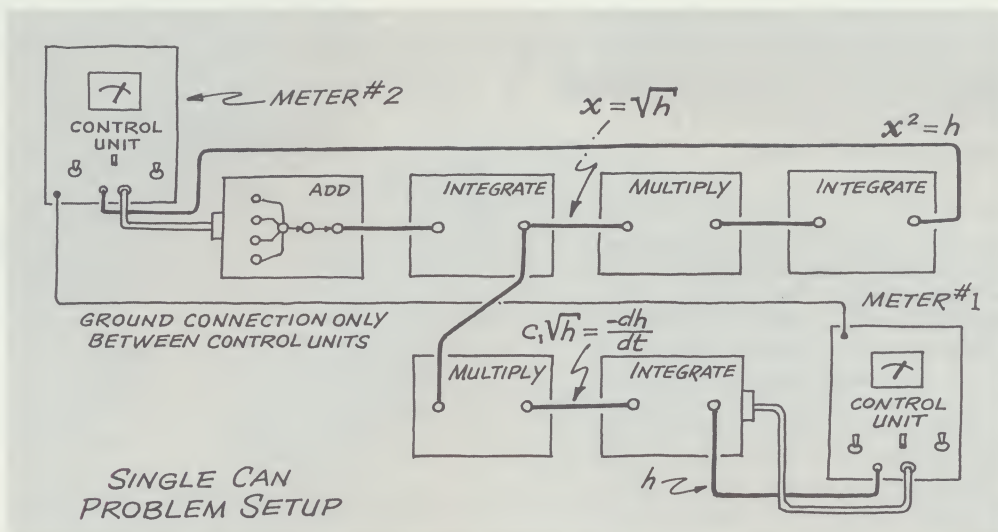


Figure 5

Operation

1. Set up the Square-Root computer as described above.
2. Set the Coefficient $C_1 = \frac{A_2}{A_1} \sqrt{2g}$
3. Set the height Initial Condition.
4. Run the Can problem for $\frac{1}{4}$ second.
5. Run the Square-Root computer until x^2 on Meter #2 equals h on Meter #1.
6. With Meter #1, measure both h and $-dh/dt$ and plot the results.
7. Repeat steps 4 through 6.

This incremental solution appears to be exactly what a digital computer does to solve problems. And so it is; however, we would have many more steps. By setting our Can Coefficient C_1 so that the can

does not empty so fast, we really do nothing more than stretch out the time scale. Let us halve the input to the Can Integrator and halve our time scale so we can have twice as many increments.

Figure 6a and 6b shows that this results in a slightly longer time for the can to empty. It becomes evident that we have difficulty in getting accuracy down at small values of the square root.

Interpretation of the Solution

The plots of the solutions show that the flow variable changes linearly reaching zero at a definite time. We should compare this with the discharge of a capacitor. The capacitor current is proportional to its voltage, so it never completely discharges! A can flow is proportional to the square root of its head.

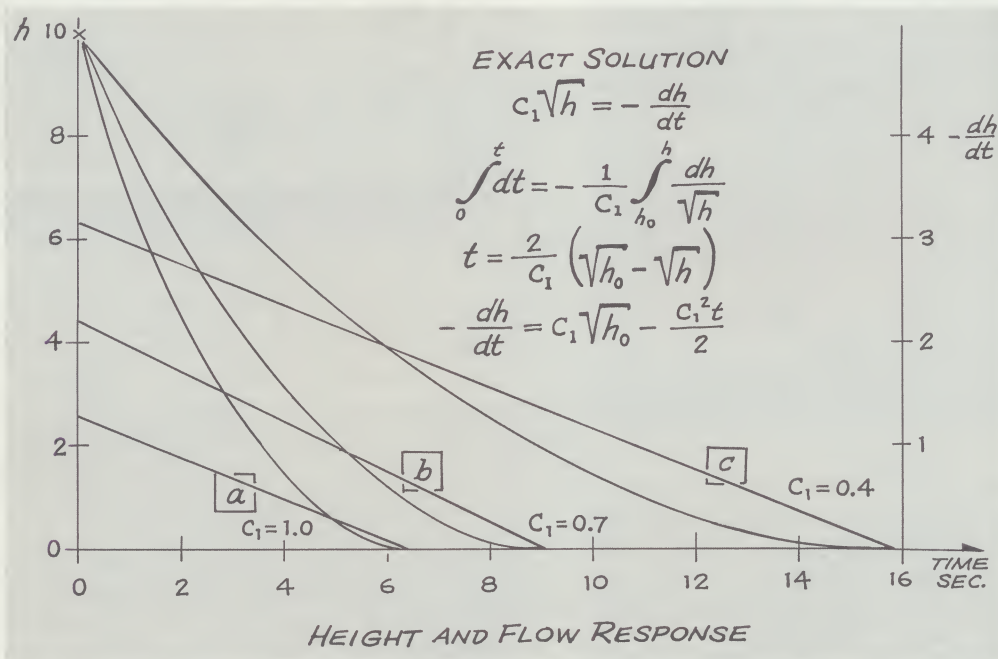


Figure 6

Although the flow decreases with height, eventually the two variables h and dh/dt become less than 1, the flow continues to decrease but not as fast as the height, and the height eventually disappears. This is a subtle concept, like Zeno's paradox. It is a situation where linear thinking breaks down.

An Exact Solution

In considering the errors in our incremental solutions, it becomes evident that if only we could make the square rooter follow the height output exactly, we could have a perfect solution. Adjusting the can constant, it is possible to have a square root output which follows the height output. This must be possible, because we know that the flow input to the can is linear (if there is an incoming flow), the water height is the integral of a linear signal, and the square-root computer is made up of linear elements. Another way of looking at it is to notice in Figure 5 that $\sqrt{h}$ is followed by a Coefficient Multiplier and Integrator in both the square-root computer and in the height computer. We only have to adjust constants to have a perfect solution with a continuous square rooter (Figure 6c).

A Homework Problem

One unfortunate aspect of this problem is that it can easily be solved by analytical techniques, since this non-linear equation happens to be easily integrated. Part of the purpose of this exercise is to tame the arrogance of students who have mastered linear techniques and who derogate the analog computer as an unnecessary substitute for pencil and paper. The following problem will not yield to an analytical solution so easily.

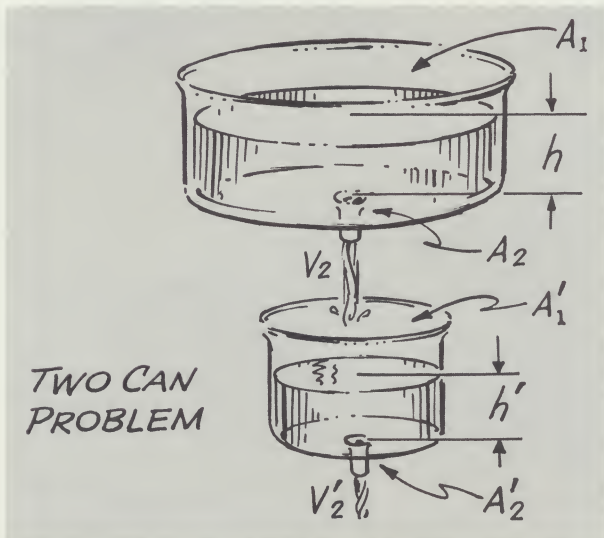
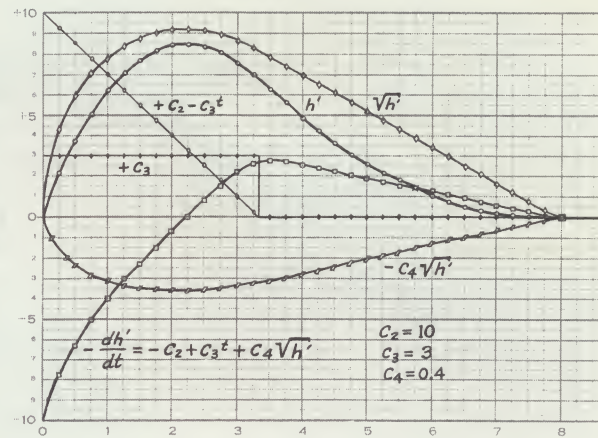


Figure 7

Suppose that a second can is placed under the first can, but it is smaller in diameter so that there is some danger of its overflowing as the first can empties into it. Determine what height you can fill the top can to and not overflow the bottom. Figure 8 shows some results from the Personal Analog Computer. For this problem we recommend analog computing, not pencil and paper.

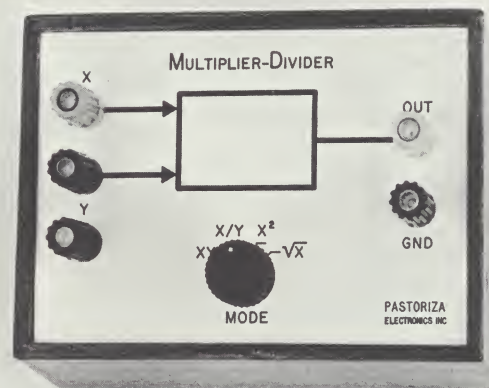


PAC SOLUTION
for TWO CAN PROBLEM

Figure 8

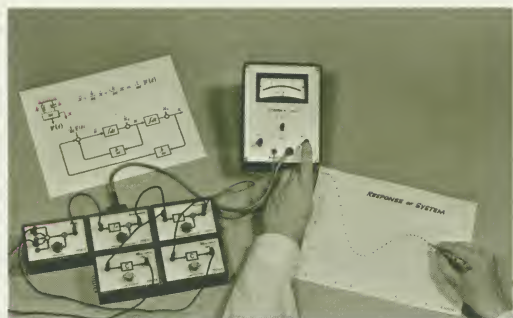
New Product Announcement

The solutions illustrated in Figure 8 for the two-can problem were made with the new Multiplier-Divider. This unit is a general-purpose non-linear operator with four modes of operation: multiplication, division, squaring, and square rooting.



The input ranges are -4 to $+4$ volts representing -1 to $+1$, and the output range is also -4 to $+4$ volts. The unit multiplies in 4 quadrants, divides in 2 quadrants (it cannot divide by negative numbers), and takes the square root of positive numbers only.

The Multiplier-Divider has an accuracy of better than $\pm 1\%$ from full scale down to output values of about 20 millivolts and will operate at frequencies up to 1,000 cps without appreciable phase shift. It requires a ± 15 volt power supply, but it is otherwise compatible with PAC. The side plugs permit direct connection into the PAC units.



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THE OPERATIONAL AMPLIFIER

A Fundamental Approach to Electronic Circuit Design

Some Dynamics History

A great upheaval in scientific history took place when the Roman numerals were abandoned in favor of Arabic Numerals (invented by the Hindus). In spite of the opposition of scholars and educated people, the Arabic system was eventually adopted throughout western Europe, and even England. The Roman System, considered adequate for ten centuries, and in which scholars had a vested interest, had tradition in its favor. The forces of change were very strong indeed.

This change came about not because of the overwhelming arguments of reasonable men. Nor did a younger generation unfettered by tradition and full of renaissance spirit revolt against it. The real factor in the change was the sudden need for mathematics in a large number of other occupations outside the mathematical discipline. People involved in commerce, shipping, navigation, banking, surveying, and a variety of other occupations came to have a very real need of a number system that could be used easily and accurately. They forced the system on the academic world.

There have been other instances of the world forcing new and more efficient methods upon pedagogical circles. Old fashioned forms of writing, German script for example, have all but disappeared. Chaucerian sentence structure and spelling have given way to more terse and direct forms of communication. Over the last two hundred years with everyone learning to read and write, the average sentence length has shrunk by about 50%.

Perhaps a recent example has been the adoption of the operational amplifier for electronic circuit work. Almost unknown 15 years ago, the operational amplifier now outsells all others. This has come about because so many people are beginning to use electronics, and they don't have the time to design individual vacuum tube or transistor circuits. With the operational amplifier as their basic active element, they can easily put together a circuit which can be completely specified, with little or no electronic background. With the advent of integrated circuits, this tendency to use modules of this type rather than individual transistors will be intensified.

What is An Operational Amplifier?

The Operational Amplifier is an amplifier designed specifically to be fed back. The feedback principle, used for many years in servo mechanisms to achieve accurate control, was incorporated into vacuum tube amplifiers to achieve superior linearity. Negative feedback reduced the dependence of gain on the amplifier characteristics. Ultimately it was found that the amplifiers could be built with almost infinite gain, and the feedback system almost entirely determining the input/output characteristics.

The infinite gain amplifier became known as an Operational Amplifier. Its symbol is shown in Figure 1. "Infinite" Gain doesn't mean that the output is infinite, but that the input is Zero. Therefore the output acts through the feedback in a way that makes this input zero. A derivation of this relationship is shown adjacent to Fig. 1, and assumes that the amplifier gain is negative infinity at all frequencies.

For our study of the applications of the Operational Amplifier we will assume that A is infinite, and the summing point voltage and current therefore zero. This greatly simplifies understanding and using the device. Such circuits as that shown in Figure 1 have great advantages.

1. The input/output relationship is independent of the amplifier characteristics and can be specified by the specifications of the feedback impedances.

2. The current drawn at the output of the amplifier has no effect on the output voltage, which must be maintained so that the summing point is zero. The output is therefore effectively a voltage source at zero impedance.

3. The input current depends only on Z_1 and is therefore isolated from the output voltage. Signal direction is one way.

4. Since the input/output relationship depends on impedances, it is possible to use a capacitor whose impedance is a derivative. Therefore one can make differentiators, and integrators, and other complex devices by using complex feedback networks.

This last feature is of course the basis of analog computers which solve differential equations.

We have shown a table which illustrates some

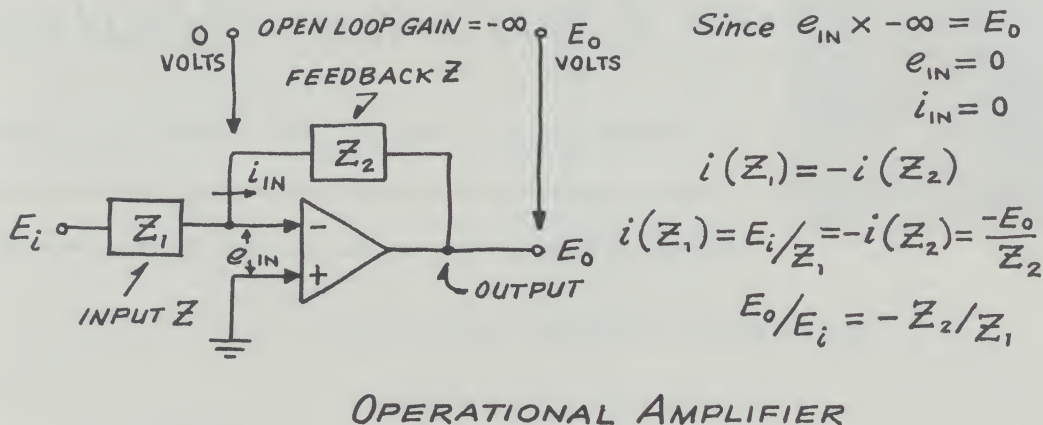


Figure 1

of the possible circuits one can make with Operational Amplifiers. The behaviour of such a circuit is relatively easy to determine. It is always the ratio of the input and feedback transfer impedances. For one not at home with transfer impedances, you need merely ask yourself, what current is the input voltage causing at the summing point? And what voltage is required at the output to draw away this current.

Real Operational Amplifiers and their Limitations

At the present time the ideal operational amplifier hasn't been designed, and there are a variety of amplifiers having various strong points to choose from for design purposes.

The major problems are non-infinite gain, and noise.

Low-gain problems occur both with low voltage gain and low current gain. Assuming low voltage gain, there must be some voltage at the summing point if there is to be an output voltage. For example with a gain of 1,000 and an output of 1 volt, one must have 1 millivolt at the input. Now if you are running with identical input and feedback impedances, i.e. at a gain of 1, the output will have to be two millivolts different from the input to make the summing voltage 1 millivolt.

A similar situation exists for current. If current is going into the summing point of the amplifier, the output voltage must adjust to provide this current. The adjustment depends on the feedback resistance. This fact indicates that lower impedances are desirable with transistor amplifiers which draw current.

Frequency Response

The Frequency response problem is a result of the loss of amplifier gain with frequency. Although an amplifier gain may be specified as 1,000,000 this is probably only at very low frequency. An Operational Amplifier will begin to lose gain at a rate of 6 db per octave eventually, and will reach unity gain anywhere between 100 KC and 100 Mc. In comput-

ing expected error, it is well to take the highest frequencies of interest into account.

Noise

An Operational Amplifier has internal signals which show up because of the efforts to get infinite gain. These signals are thermal noise effects, and logically extend down to d.c. where they are called drift with temperature. They are an important specification of the amplifier, because they represent the basic resolution and accuracy you can achieve with the circuit. The "noise gain" of the circuit you design is how much you will amplify the noise in the input of the amplifier. Books are written on noise, and it must suffice here to remind the user of its existence.

Conclusion:

The academic world is no longer the last to accept and adopt new techniques. Since schools and industry have begun to share their employees, students, and problems, the universities now lead the rest of the world into new methods.

The basic nature of the Operational Amplifier, and its importance as an engineering tool in every discipline has been recognized by Dartmouth and resulted in their specifying the design of the Personal Amplifier Manifold. They wanted the portability and convenience of PAC, but the greater flexibility of uncommitted Operational Amplifiers to which the student could fit his own feedback networks. They required high performance amplifiers, and regulated power supplies. The amplifiers were removable so that different types could be used.

Some typical PAM circuits are: Waveform generators, triangle, sine and square waves, multi-vibrators, operators, integrators, differentiators, leads, lags, low high and band pass filters, multiple pole networks, analog computers, general purpose amplifiers, differential amplifiers, servo systems, control loop simulation, plant simulation, solutions of matrix and linear algebraic problems.

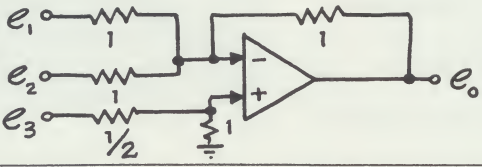
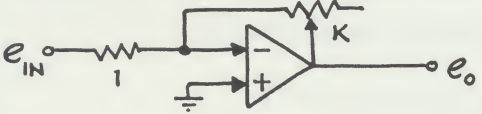
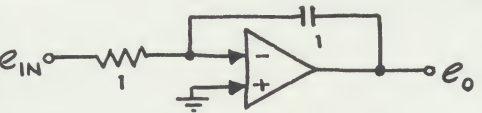
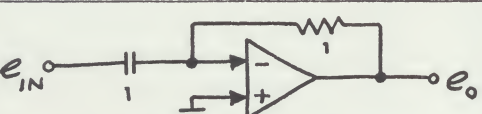
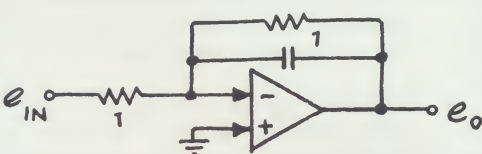
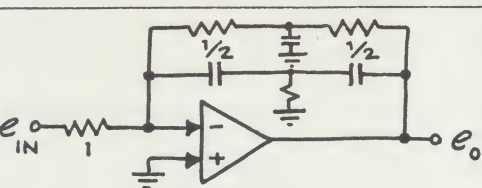
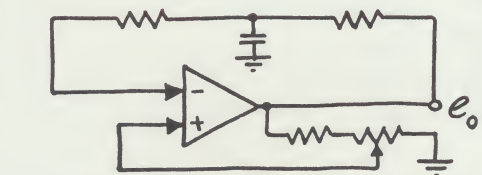
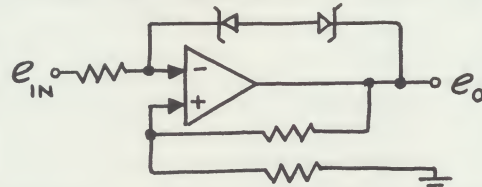
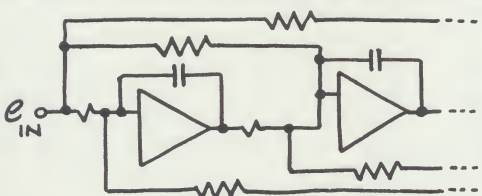
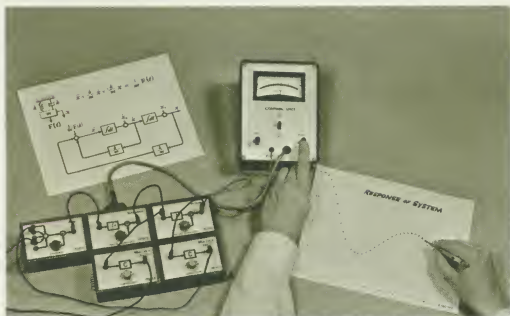
Some Useful Applications of Operational Amplifiers			
NAME	CONFIGURATION	MATHEMATICAL RELATIONSHIP	APPLICATION FEATURES
Adder and Subtractor		$e_3 - e_1 - e_2 = e_o$	Single Amplifier sums and subtracts many signals.
Linear Coefficient Multiplier		$e_o = -K e_{IN}$	Linearly variable gain with low output impedance.
Linear Integration		$e_o = -\int_0^t e_{IN} dt$	A very simple way to integrate.
Differentiator		$e_o = -\frac{de_{IN}}{dt}$	For sensing changes.
Lag		$e_o = \frac{1}{1 + T_1 s} e_{IN}$	Approximates a running time average.
Filter		$e_o = \frac{1}{1 + T_1 s + T_2^2 s^2}$	Narrow band filter.
Oscillator		$e_o = \frac{1}{1 + T_2 s^2}$	Simple low frequency oscillator.
Schmitt Trigger		$e_o = +1 \quad e_{IN} < -a$ $e_o = -1 \quad e_{IN} > a$	Determines zero crossings.
Multiple Pole Systems		$\frac{a_1 + a_2 s + a_3 s^2}{b_1 + b_2 s + b_3 s^2}$	Simulating complex dynamic systems. Custom filters.

Table 1



The Dynamic Analyzer.

A periodical devoted to applications of the Personal Analog Computer.

Published by Pastoriza Electronics, Inc. 285 Columbus Ave., Boston 16, Mass.

LOGARITHMS

Introduction

We have recently finished a study on logarithms using the PAC units at the request of a prominent local prep school. These notes are available upon request. Rather than hire a high priced consultant to pontificate in this issue on exponentials and logs, we have decided to publish some recent letters more edifying than our run-of-the-mill correspondence.

Letters to the Editors:

Dear Sirs,

April 1, 1964

Thank you for the loan of Your PAC equipment. We fully intend to purchase two units in the near future, but like most schools suffer from "Pedagogic Purse." We have had no difficulties with the unit except for running down the batteries. I am following your suggestion about turning them off in the future.

In one of your earlier issues of the *D/A* you suggested four methods of extracting the square root of 2. I was astonished to find that you neglected one of the most instructive and practical techniques illustrated in the accompanying sketch. I allow full scale to represent 2. Thus 5 on the scale represents 1. Starting my Integrator at the initial condition of 1, I let it expand exponentially for an even number of $\frac{1}{4}$ second increments until it exactly reaches 2. This takes some adjusting of C . With C set at about $-.68$ the exponential will grow from one to two in exactly 1 second. This means that it will grow to the square root of 2 in $\frac{1}{2}$ second. (7.07 on the scale). You can also find the fourth root. Do you know the significance of the $.68$ setting for C ?

Please continue to send me the *D/A*. I enjoy its amateur pretentiousness, even though it is full of errors. Who writes it?

Sincerely yours,

H. Langdon Beasley Jr.

Dear Mr. Beasley,

The $.68$ is the log to the base e of 2. We use a high priced consultant on *D/A* (not suffering from "Pedagogic Purse"). Hope you get money soon. Have you tried the government? Write again.

Editor

April 15, 1964

Gentlemen,

I have noted a number of statements in the press comparing your PAC equipment to an electronic slide rule. Those fellows are trying to arouse interest by attaching the glamour word "electronics" onto the only analog computer anyone knows about, the slide rule. I can see no analogy. Isn't this a false name? Aren't you obligated to discourage the analogy?

Yours,

A Student

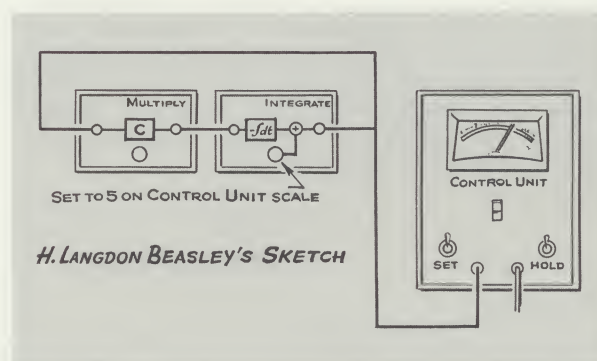


Figure 1

Dear Student,

In answer to your two questions, No, No.

The slide rule is a mechanical representation of a variable and its logarithm. It is a compact form of the graph in Figure 2. The abscissa represents length on the slide rule, and the ordinate the numbers printed along that length. In Figure 3 alongside we show the relationship between the Integrator output in Beasley's set up of Fig. 1, and time. The analogy is perfect in this case. Time and voltage readings represent length and printed readings on the slide rule respectively. It would be difficult to think of a better electronic "slide rule" analogy.

Editor

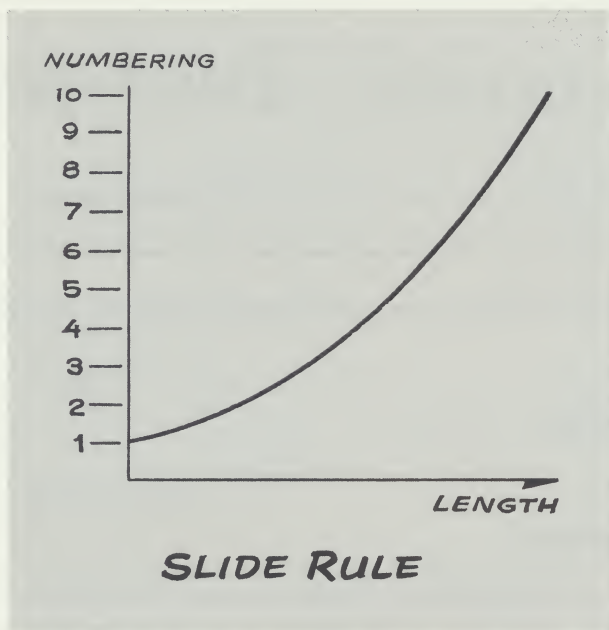


Figure 2

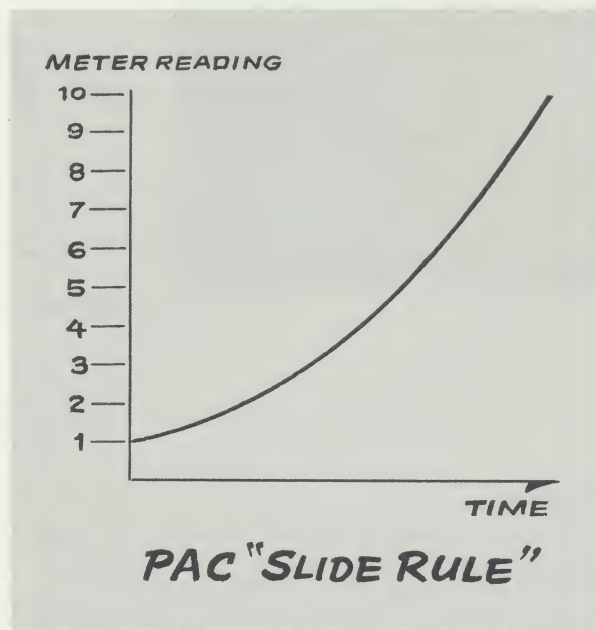


Figure 3

April 3, 1964

Gentlemen,

I have purchased one of your PAC computers, and aside from battery troubles, it has worked very well. I have been trying to impress my colleagues (who have IBM equipment) by computing the square root of 2 as suggested in your issue #5. However they point out that this can be done with compass and straight edge, said equipment being somewhat less expensive. Can you suggest some other problem not so easily solved?

Sincerely,
Elsworth Oglethorpe.

Dear Mr. Oglethorpe,

I suggest computing π or e , both of which are transcendental numbers. e can be computed from the Beasley computer shown in Figure 1, allowing the exponential growth from an initial condition of 1 at a loop gain of 1 (C set to -1) for exactly 1 second. The value will then be e .

π can be generated by making a sine wave generator, i.e. two integrators and a negative coefficient with gain of -1 . Measure how long one cycle takes. I also suggest you brush up on what transcendental numbers are before confronting your colleagues.

Sincerely,
Editor

ADVANCE NOTICE

Again this year, the *PERSONAL ANALOG COMPUTER* will be at the American Society for Engineering Education Annual Meeting to be held at the University of Maine, June 22 through 25, 1964.

THE PERSONAL ANALOG COMPUTER

Instruments for Instruction and Training

The concept of a Personal Analog Computer was proposed and specified at Case Institute of Technology by Professor James Reswick. Realizing that the use of analogue techniques greatly strengthened the students knowledge of physical systems, as well as their associated mathematics, he suggested the development of a "dormitory computer" specifically for student use. Each student would have his own portable unit, to use when and where he wanted to experiment. The use of the computer would require no special knowledge. The culmination of this proposal was the Personal Analog Computer.

What is the Personal Analog Computer

The Personal Analog Computer is an electronic demonstration system which utilizes individual miniature analog computer modules for visually representing and solving mathematical equations. Each of these modules performs one of the three basic mathematical operations, adding, multiplication by a constant, and integration.

By appropriately interconnecting these units and adjusting their parameters, a variety of complex physical systems may be simulated.

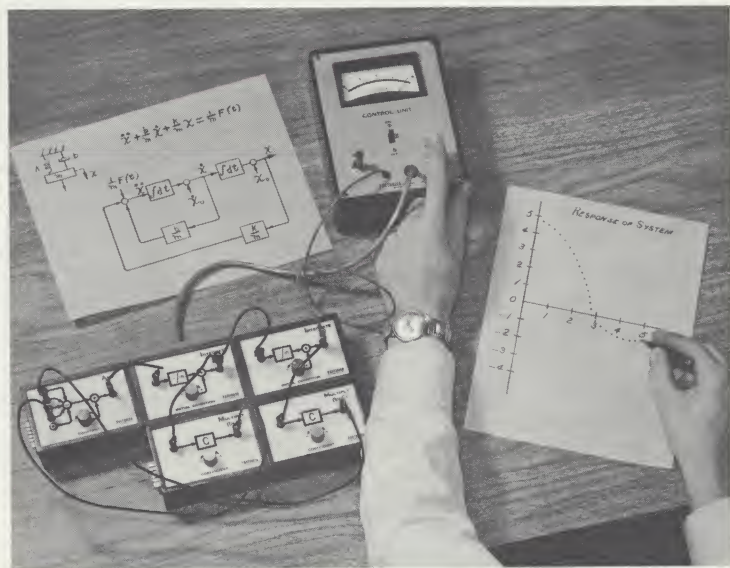
Behaviour of system variables can be monitored and plotted by the student.

Why was PAC Developed

The modular PAC (Personal Analog Computer) system was developed to provide students with a model intermediate between a dynamic physical system and the mathematical representation of that system.

After a student learns the physical features of a system, and the differential equations governing the system, he can interconnect PAC units to represent that system (or the equation). The Units will duplicate in real time the behaviour of the system variables.

PAC units provide the student with a personal experience associating physical behaviour with the solutions of a differential equation. They permit him to test his knowledge on a direct model personally at any time and in any place. This experimentation will reinforce both his physical and mathematical understanding.



Specifications

There are four PAC modules: ADDER, INTEGRATOR, COEFFICIENT MULTIPLIER, and the CONTROL UNIT.

Accuracy 1%

Panels Each module has a graphic panel face which indicates its mathematical operation. Panels are white and take crayon easily for marking purposes.

Interconnection Units are physically and electrically connected by four gold plated connectors. These carry power and control signals.

Variables Variables are interconnected by miniature banana patch cords.

Parameters Each unit has a front panel control to adjust its parameters.

Power Portable power is contained in the Control Unit.

Read Out Large center position precision meter, permits the measurement of any variable.

Figure 1; shows a typical physical system that a student might want to analyze, that of a spring and mass.

The parameters are the spring constant and the mass is assumed to be 1 for simplicity. The variables are x , displacement and f , force between the mass and the spring.

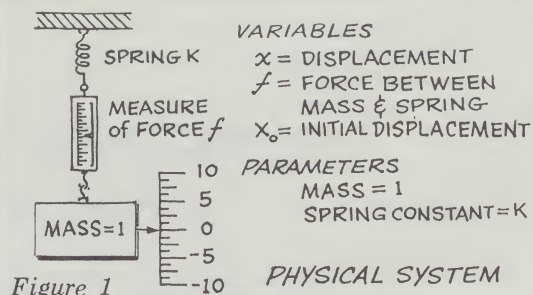


Figure 1

Figure 2; shows the differential equations that he would write showing the initial position of the mass displaced an amount X_0 .

Figure 2

$$-kx = \frac{d^2x}{dt^2} \quad \begin{matrix} t = 0 \\ x = x_0 \\ dx/dt = 0 \end{matrix}$$

DIFFERENTIAL EQUATION

Figure 3; shows the interconnection of PAC units. Each patch cord is a variable. Each box is a mathematical operation. The student sets the Coefficient Multiplier to the spring constant, the middle integrator to X_0 as an initial condition, and the first Integrator to zero, meaning that the initial velocity is zero.

Pushing the SET switch on the Control Unit sets up this initial situation. Pushing the HOLD switch down starts the problem running and the X variables will follow the sine wave shown in the graph. Pushing the HOLD switch up causes the problem to run in $1/4$ second intervals.

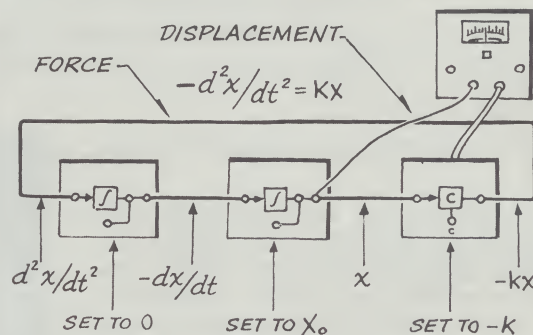


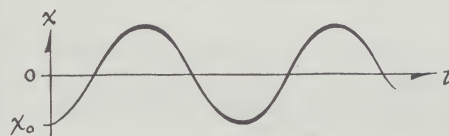
Figure 3

PAC UNITS

Figure 4; whenever the student puts the HOLD switch in the neutral position, the problem is frozen and reading and plotting of variables can take place. This then is the resulting graphic solution.

Figure 4

GRAPHIC SOLUTION



The Integrator Unit, a typical PAC element

The integrator Unit is shown in Figure 6 without cover to indicate the packaging and precision components. Note particularly:

- Precision Mylar condenser for integrating current
- Two high-speed sealed reed relays for control
- All printed-circuit transistorized construction
- Large, high resolution potentiometer
- Miniature transistorized operational amplifier

The standard complement of PAC units consist of two Integrators, two Coefficient Multipliers, an Adder, and a Control Unit. Aside from the ordinary first and second order differential equations, PAC units are useful in demonstrating nonlinear differential equation solution, nonlinear circuits such as flipflops, multivibrators, waveform generators, and a number of interesting numerical experiments.

For further information or advice on applications, write or phone

PASTORIZA ELECTRONICS, INC.

285 Columbus Avenue Boston 16, Massachusetts

Telephone: COMmonwealth 6-1918

